1 Basic Concepts

This chapter develops a number of basic motor concepts in a way that appeals to your intuition. In doing so, the concepts are more likely to make sense, especially when these concepts are used for motor design in later chapters. Many of the concepts presented here apply to most motor types since all motors are constructed of similar materials and all produce the same output, namely torque.

1.1 Scope

This text covers the analysis and design of rotational brushless permanent magnet (PM) motors. Brushless DC, PM synchronous, and PM step motors are all brushless permanent magnet motors. These specific motor types evolved over time to satisfy different application niches, but their operating principles are essentially identical. Thus, the material presented in this text is applicable to all three of these motor types, with particular emphasis given to brushless DC and PM synchronous motors.

To put these motor types into perspective, it is useful to show where they fit in the overall classification of electric motors as shown in **Fig. 1-1**. The other motors shown in the figure are not considered in this text. Their operating principles can be found in a number of other texts.

Brushless DC motors are typically characterized as having a trapezoidal back electromotive force (*i.e.*, back EMF) and are typically driven by rectangular pulse currents. This mimics the operation of brush DC motors. From this perspective, the name “brushless DC” fits even though it is an AC motor. PM synchronous motors differ from brushless DC motors in that they typically have a sinusoidal back EMF and are driven by sinusoidal currents. Step motors in general have high pole counts and therefore require many periods of excitation for each shaft revolution. Even though they can be driven like other synchronous motors, they are typically driven with current pulses. Step motors are typically used in low cost, high volume, position control applications where the cost of position feedback cannot be justified.

1.2 Shape

The most common motor shape is cylindrical as shown in **Fig. 1-2**. This motor shape and all others contain two primary parts. The nonmoving or stationary part is called the stator. The moving or rotating part is called the rotor. In most cyl-

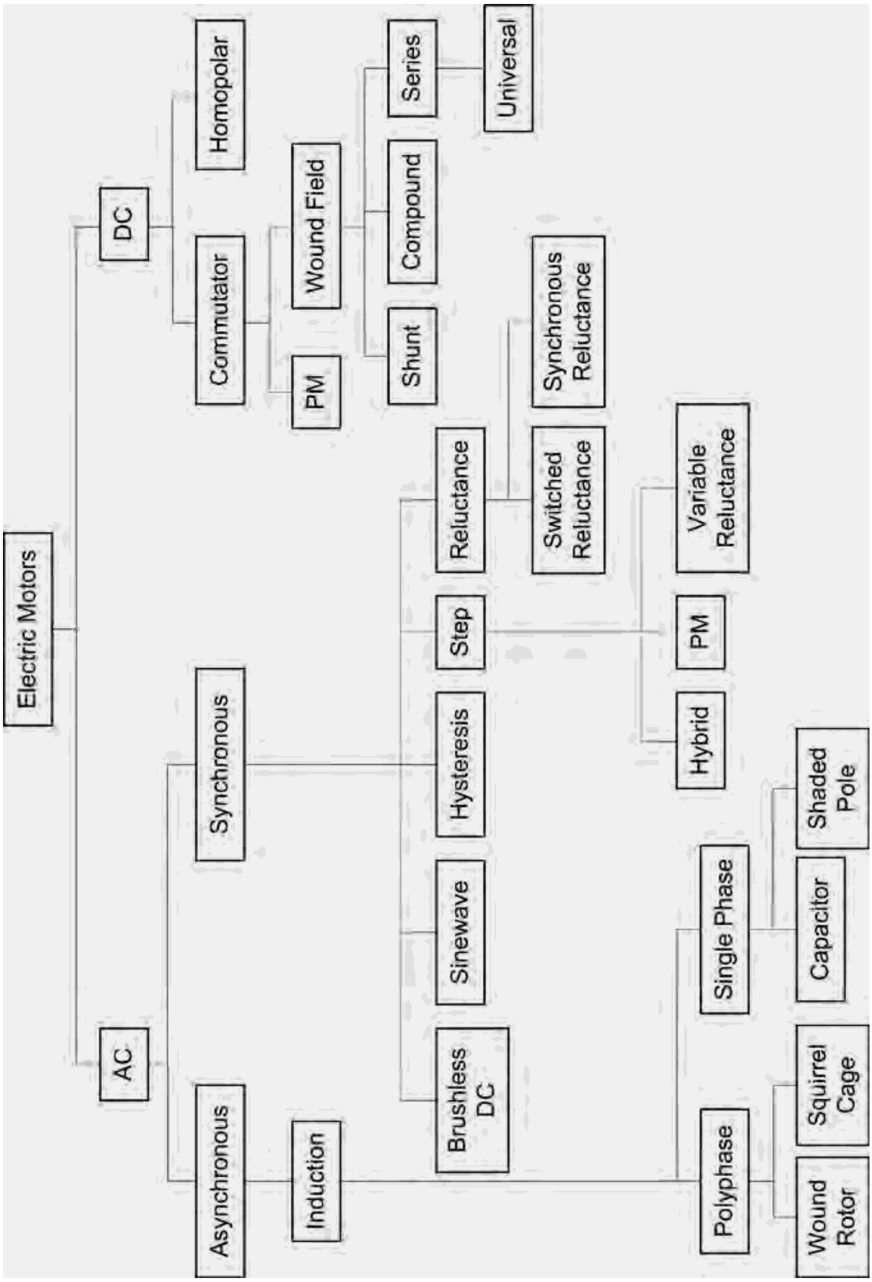

Figure 1-1. A classification of motors.

indrical motors, the rotor appears inside the stator as shown in Fig. 1-2*a*. This construction is popular because placing the nonmoving stator on the outside makes it easy to attach the motor to its surroundings. Moreover, confining the rotor inside the stator provides a natural shield to protect the moving rotor from its surroundings.

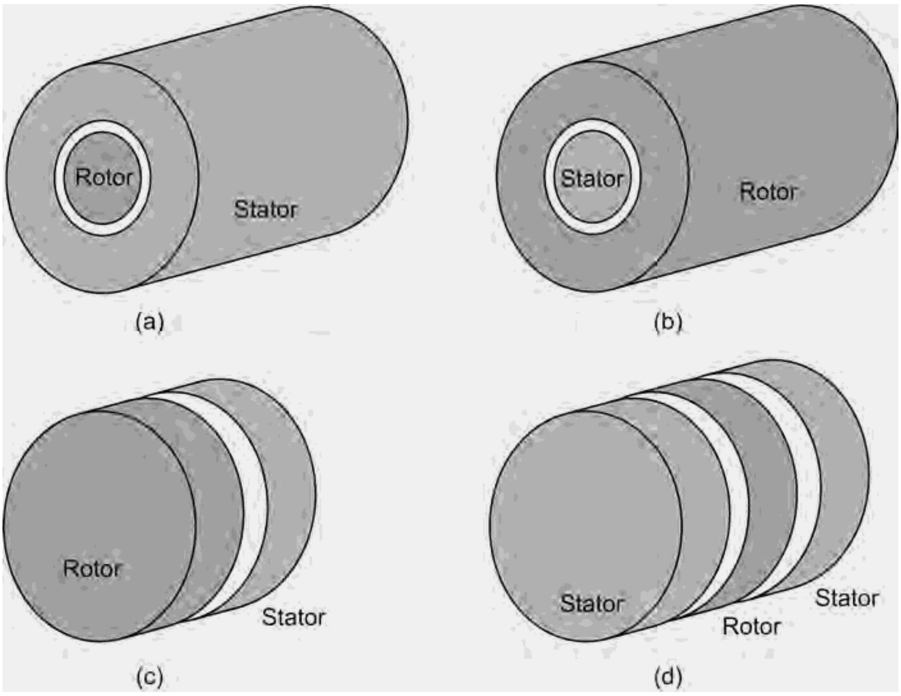

Figure 1-2. Some motor construction possibilities.

In addition to the cylindrical shape, motors can be constructed in numerous other ways. Several possibilities are shown in Fig. 1-2. Figs. 1-2*a* and 1-2*b* show the two cylindrical shapes. When the rotor appears on the outside of the stator as shown in Fig. 1-2*b*, the motor is often said to be an inside-out motor. For these motors, a magnetic field travels in a radial direction across the air gap between the rotor and stator. As a result, these motors are called radial flux motors. Motors having a pancake shape are shown in Figs. 1-2*c* and 1-2*d*. In these axial flux motors, the magnetic field between the rotor and stator travels in the axial direction.

Brushless PM motors can be built in all the shapes shown in Fig. 1-2, as well as in a number of other more creative shapes. All brushless PM motors are constructed with electrical windings on the stator and permanent magnets on the rotor. This construction is one of the primary reasons for the increasing popularity of brushless PM motors. Because the windings remain stationary, no potentially troublesome moving electrical contacts, *i.e.*, brushes are required. In addition, stationary windings are easier to keep cool.

The common cylindrical shape shown in Fig. 1-2, leads to the use of the cylindrical coordinate system as shown in **Fig. 1-3**. Here the r-direction is called radial, the z-direction is called axial, and the θ-direction is called tangential or circumferential.

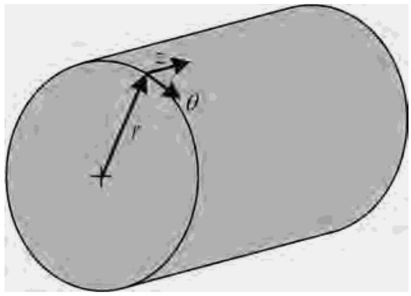

Figure 1-3. The cylindrical coordinate system.

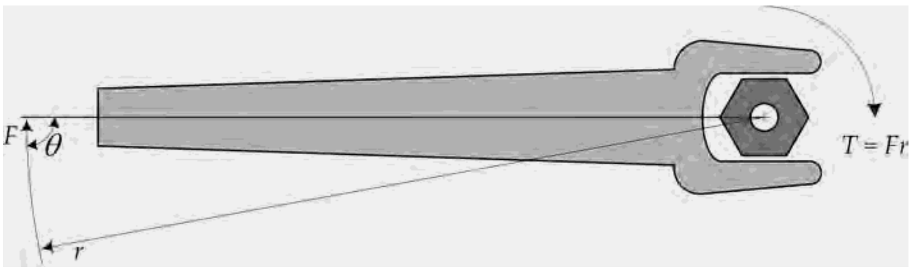

Figure 1-4. A wrench on a nut.

1.3 Torque

All motors produce torque. Torque is given by the product of a tangential force and the radius at which it acts, and thus torque has units of force times length, *e.g.*, ozf·in, lbf·ft, or N·m. To understand this concept, consider the wrench on the nut shown in **Fig. 1-4**. If a force F is applied to the wrench in the tangential direction, *i.e.*, perpendicular to the handle, at a distance r from the center of the nut, the twisting force or torque experienced by the bolt is

$$T = F\,r \tag{1.1}$$

This relationship implies that if the length of the wrench is doubled and the same force is applied at a distance $2r$, the torque experienced by the nut is doubled. Likewise, shortening the wrench by a factor of two and applying the same force cuts the torque in half. Thus, a fixed force produces the most torque when the radius at which it is applied is maximized. Furthermore, it is only force acting in the tangential direction creates torque. If the force is applied in an outwardly radial direction, the wrench simply comes off the nut and no torque is experienced by the nut. Taking the direction of applied force into account, torque can be expressed as $T = F r \sin(\theta)$, where θ is the angle at which the force is applied with respect to the radial direction.

This concept of torque makes sense to anyone who has tried to loosen a rusted nut: the longer the wrench, the less force required to loosen the nut. And the force applied to the wrench is most efficient when it is in the circumferential direction, *i.e.*, in the direction tangential to a circle centered over the nut as shown in Fig. 1-4. Clearly if the force is applied in an outward radial direction, the nut experiences no torque, and the wrench comes off the nut.

1.4 Motor Action

With an understanding of torque production, it is now possible to illustrate how a brushless permanent magnet motor works. All that is required is the rudimentary knowledge that magnets are attracted to steel, that opposite magnet poles attract, that like magnet poles repel each other, and that current flowing in a coil of wire makes an electromagnet.

Consider the bar permanent magnet centered in a stationary steel ring as shown in **Fig. 1-5**, where the bar magnet in the figure is free to spin about its center, but is otherwise fixed. The magnet is the rotor and the steel ring is the stator, and they are separated by an air gap. As shown in the figure, the magnet does not have any preferred resting position. Each end experiences an equal but oppositely-directed, radial force of attraction to the ring that is not a function of the particular direction of the magnet. The magnet experiences no net force, and thus no torque is produced.

Next consider changing the steel ring so that it has two protrusions or poles on it as shown in **Fig. 1-6**. As before, each end of the magnet experiences an equal but oppositely directed radial force. Now however, if the magnet is spun slowly it will have the tendency to come to rest in the aligned position at $\theta = 0°$ or $\theta = 180°$. That is, as the magnet spins it will experience a force that will try to align the magnet with the stator poles. This occurs because the force of attraction between a magnet and steel increases dramatically as the physical distance between the two decreases. Because the magnet is free to spin, the force of attraction is partly in the tangential direction and torque is produced.

Figure 1-7 depicts the torque produced by this force graphically as a function of motor position. The positions where the torque is zero are called detent positions. When the magnet is aligned with the poles, any small disturbance causes the magnet to restore itself to the aligned position. Thus these detent positions are said to

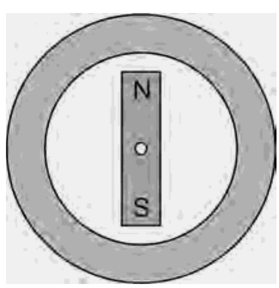

Figure 1-5. A magnet free to spin inside a steel ring.

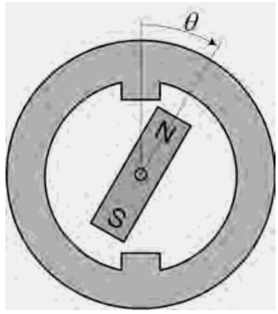

Figure 1-6. A magnet free to spin inside a steel ring having two poles.

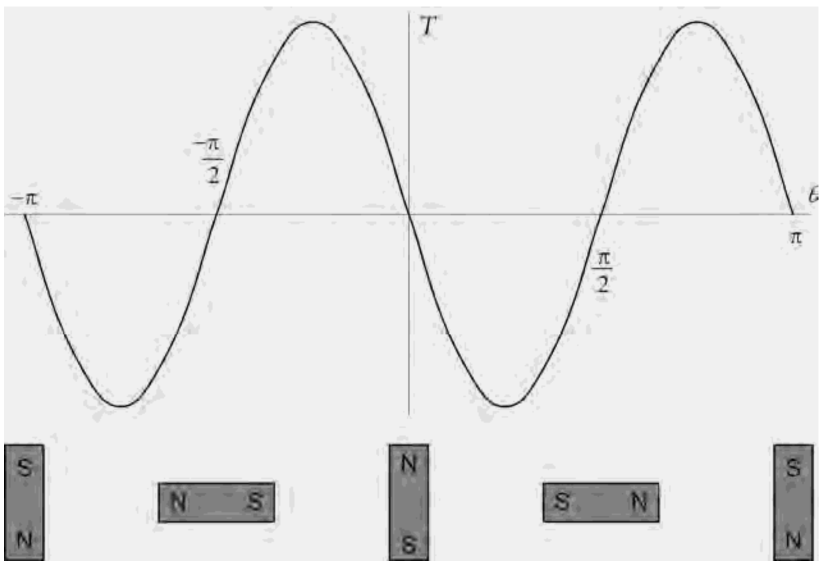

Figure 1-7. Torque experienced by the magnet in Fig. 1-6.

be stable. On the other hand, when the magnet is halfway between the poles, any small disturbance causes the magnet to move away from the unaligned position and seek alignment. Thus, unaligned detent positions are said to be unstable. While the shape of the detent torque is approximately sinusoidal in Fig. 1-7, in a real motor its shape is a complex function of motor geometry and material properties. The torque described here is formally called reluctance torque, and more commonly cogging torque. In most applications, cogging torque is undesirable.

Now consider the addition of current carrying coils to the poles as shown in **Fig. 1-8**. If current is applied to the coils, the poles become electromagnets. In particular, if the current is applied in the proper direction, the poles become magnetized as shown in Fig. 1-8. In this situation, the force of attraction between the bar magnet and the opposite electromagnet poles creates another type of torque, formally called mutual or alignment torque. It is this torque that is used in brushless PM

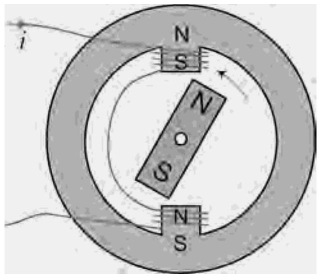

Figure 1-8. Current-carrying windings added to Fig. 1-6.

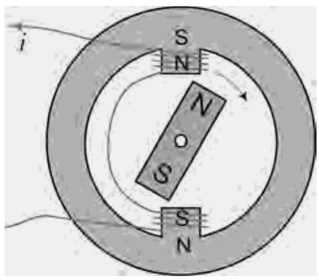

Figure 1-9. Current flow in the opposite direction compared to Fig. 1-8.

motors to do work. The term mutual is used because it is the mutual attraction between the magnet poles that produces torque. The term alignment is used because the force of attraction seeks to align the bar magnet and coil-created magnet poles.

This torque could also be called repulsion torque, since if the current is applied in the opposite direction, the poles become magnetized in the opposite direction as shown in **Fig. 1-9**. In this situation the like poles repel, sending the bar magnet in the opposite direction. Since both of these scenarios involve the mutual interaction of the magnet poles, the torque mechanism is identical, and the term repulsion torque is not used.

To get the bar magnet to turn continuously, it is common to employ more than one set of coils. **Fig. 1-10** shows the case where three sets of coils are used. These sets are called phase windings or simply windings. In the figure, the phases are labeled A, B, and C. The phase labels with overbars are used simply to denote where opposite magnet poles are created facing the rotor magnet. By creating electromagnet poles on the stator that attract and repel those of the bar magnet, the bar magnet can be made to rotate by successively energizing and deenergizing the phases in a process called commutation.

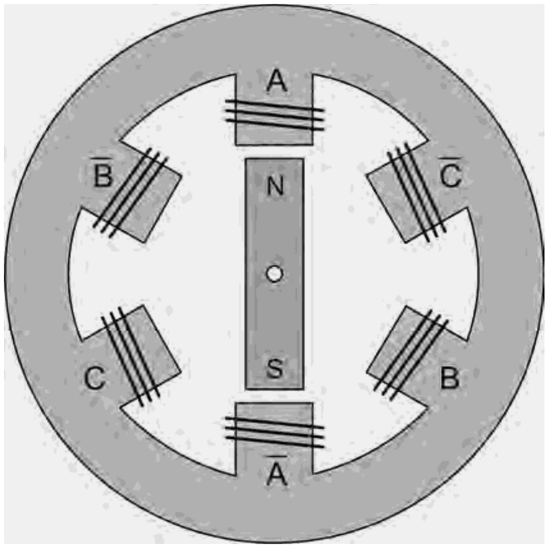

Figure 1-10. A motor having three phases.

1.5 Magnet Poles and Motor Phases

Although the motor depicted in Fig. 1-10 has two rotor magnet poles and three stator phases, it is possible to build brushless PM motors with any even number of rotor magnet poles and any number of phases greater than or equal to one. Two and three phase motors are the most common, with three phase motors dominating all others. The reason for these choices is that two and three phase motors minimize the number of power electronic devices required to control the winding currents.

The choice of magnet poles offers more flexibility. Brushless PM motors have been constructed with two to fifty or more magnet poles, with the most common being single digit values. As will be shown later, a greater number of magnet poles usually creates a greater torque for the same current level. On the other hand, more magnet poles means less space for each pole. Eventually, a point is reached where the spacing between rotor magnet poles becomes a significant percentage of the total room on the rotor, and torque no longer increases. The optimum number of magnet poles is a complex function of motor geometry and material properties.

1.6 Poles, Slots, Teeth, and Yokes

The motor in Fig. 1-10 has concentrated or solenoidal windings. That is, the windings of each phase are isolated from each other and concentrated around individual poles called salient poles in much the same way that a simple solenoid is wound. A more commonly occurring alternative to this construction is to use distributed windings where the windings of each phase overlap as shown in **Fig. 1-11**. The stator now has teeth that protrude toward the magnets on the rotor from an

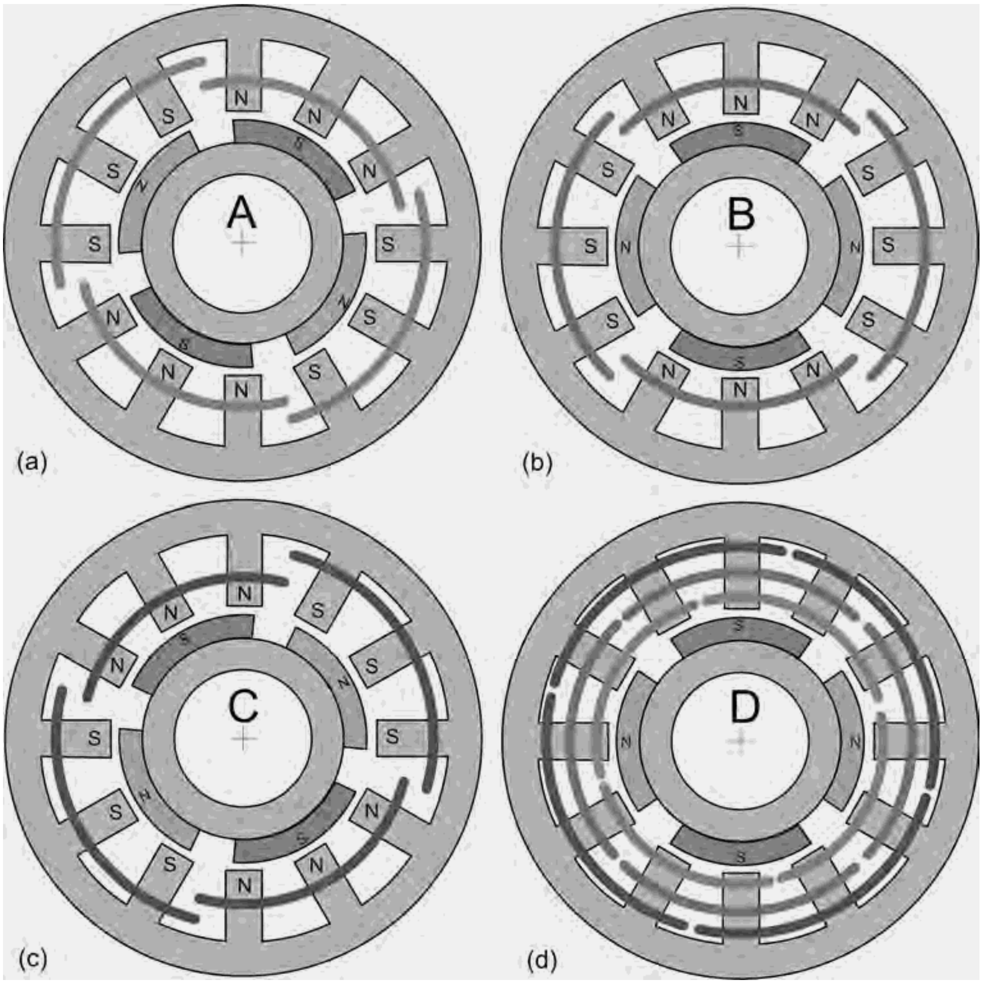

Figure 1-11. A motor with distributed windings.

outer ring of steel called the stator yoke or back iron. In between the teeth are slots that are occupied by the windings. Each winding travels from one slot, across a number of teeth (three in this case), then down the next slot. The teeth enclosed by a winding forms the pole for that coil. When phase windings are energized individually, the rotor rotates into alignment with the associated magnetic poles created on the stator. Figs. 1-11*a*, *b*, and *c* show the motor with the isolated windings for phases A, B, and C respectively. The figures also show the magnetic poles formed at the tooth tips when each phase winding is energized. Again, the sequential energization of phase windings causes the rotor to rotate. To keep things visually simple, Fig. 1-11*d* illustrates the completely wound motor. When all coils are put in place, the wound motor has two coil sides in each slot. While not true for many motors, each slot in this motor contains two coil sides from the same phase. There are no slots with coil sides from different phases.

The rotors depicted in Fig. 1-11 are formed from circular arc shaped magnet pieces attached to an inner ring of steel call the rotor yoke or back iron. The magnets are magnetized in alternating directions as one proceeds around the rotor periphery. In this case, the rotors have four magnet poles as opposed to two magnet poles shown in Fig. 1-10.

The motor cross sections shown in Fig. 1-11 are more representative of actual motors than those shown earlier, but they are still much simpler than real motors. In later chapters, a variety of more practical motor construction details will be presented.

1.7 Mechanical and Electrical Measures

In electric motors it is common to define two related measures of position and speed. Mechanical position and speed are the respective position and speed of the rotor shaft. When the rotor shaft makes one complete revolution, it traverses 360 mechanical degrees (°M) or 2π mechanical radians (radM). Having made this revolution, the rotor is right back where it started.

Electrical position is defined such that movement of the rotor by 360 electrical degrees (°E) or 2π electrical radians (radE) puts the rotor back in an identical magnetic orientation. In Fig. 1-10, mechanical and electrical position are identical since the rotor must rotate 360 °M to reach the same magnetic orientation. On the other hand, in Fig. 1-11 the rotor need only move 180 °M to have the same magnetic orientation. Thus, 360 °E is the same as 180 °M for this case. Based on these two cases, it is easy to see that the relationship between electrical and mechanical position is related to the number of magnet poles on the rotor. If N_m is the number of magnet poles on the rotor facing the air gap, *i.e.*, $N_m = 2$ for Fig. 1-10 and $N_m = 4$ for Fig. 1-11, this relationship can be stated as

$$\theta_e = \frac{N_m}{2}\theta_m \tag{1.2}$$

where θ_e and θ_m are electrical and mechanical position respectively. Since magnets always have two poles, it is common to define a pole pair as one North and one South magnet pole facing the air gap. In this case, the number of pole pairs is equal to $N_p = N_m/2$ and the above relationship is simply

$$\theta_e = N_p \theta_m \tag{1.3}$$

Differentiating (1.2) and (1.3) with respect to time gives the relationship between electrical and mechanical frequency or speed as

$$\omega_e = \frac{N_m}{2}\omega_m = N_p \omega_m \tag{1.4}$$

where ω_e and ω_m are electrical and mechanical frequencies respectively in radians per second. This relationship can also be stated in terms of Hertz (Hz) as

$$f_e = \frac{N_m}{2} f_m = N_p f_m \tag{1.5}$$

where $f_m = \omega_m/(2\pi)$. Later, when harmonics of f_e are discussed, f_e will be called the fundamental electrical frequency.

It is common practice to specify motor mechanical speed S in revolutions per minute (rpm). For reference, the relationships among S, f_m, and f_e are given by

$$\omega_m = \frac{\pi}{30} S \approx \frac{S}{10} \tag{1.6}$$

$$f_e = \frac{N_m}{120} S = \frac{N_p}{60} S \tag{1.7}$$

This last equation is useful because it describes the rate or frequency at which commutation must occur for the motor to turn at a given speed in rpm. The inverse of this frequency gives the commutation time period, *i.e.*, the length of time over which the energizing of a phase completes one cycle of operation.

The fundamental electrical frequency f_e influences the design of the power electronics used to drive the motor. As f_e increases, the power electronics must act faster to keep the motor shaft turning. Acting faster makes the power electronics become more expensive as f_e increases. Because of this, it is common to use fewer magnet poles, *i.e.*, reduce N_m, for motors designed to operate at high speeds. However, reducing N_m does not come without a penalty. As the magnet pole count decreases, the torque production efficiency drops. Therefore, one must find a compromise between power electronics cost and torque production efficiency when choosing the number of magnet poles.

Variables such as θ and ω are used in this text both with and without various subscripts to denote respective positions and velocities. In some situations, these variables describe quantities in electrical measure (°E or radE); in other places they describe quantities in mechanical measure (°M or radM). The subscript e generally-denotes electrical measure; whereas the subscript m generally denotes mechanical measure. When these subscripts are not used, the context of passage where they appear clarifies their unit measure.

In addition to electrical and mechanical measures, a variety of other measures exist. A number of these appear when studying motor vibration characteristics. For example, the pole passage frequency is given in rad/s and Hz respectfully as

$$\omega_{pp} = N_m \omega_m$$
$$f_{pp} = \frac{\omega_{pp}}{2\pi} = \frac{N_m}{60} S \tag{1.8}$$

respectively. This frequency is simply twice the electrical frequency of the motor and is the rate at which a North or South magnet pole moves past any fixed point on the stator.

The slot passage frequency is simply the frequency at which the rotor rotates with respect to the number of slots N_s that the motor has. In rad/s and Hz, this frequency is

$$\omega_{sp} = N_s \omega_m$$
$$f_{sp} = \frac{\omega_{sp}}{2\pi} = \frac{N_s}{60} S \tag{1.9}$$

1.8 Motor Size

A fundamental question in motor design is: How big does a motor have to be to produce a required torque? For radial flux motors the answer to this question is often stated as

$$T = k D^2 L \tag{1.10}$$

where T is torque, k is a constant, D is the rotor diameter, and L is the axial rotor length. To understand this relationship, reconsider the motor shown in Fig. 1-10.

First assume that the motor has an axial length (depth into page) equal to L. For this length, a certain torque T_L is available. Now if this motor is duplicated, added to the end of the original motor, and the rotor shafts connected together, the total torque available becomes the sum of that from each motor, namely $T = T_L + T_L$. That is, an effective doubling of the axial rotor length to $2L$ doubles the available torque. Thus, torque is linearly proportional to L as shown in (1.10).

Understanding the D^2 relationship requires a little more effort. In the discussion of the wrench and nut shown in Fig. 1-4, it was stated that a given force produces a torque that is proportional to radius, *i.e.*, $D/2$. Therefore, torque is at least linearly proportional to diameter. However, it can be argued that the ability to produce force is also linearly proportional to diameter. This follows because the rotor perimeter increases linearly with diameter, *e.g.*, the circumference of a circle is equal to πD. A simple way to see this relationship is to compare the simple motor in Fig. 1-10 to that in Fig. 1-11. If the motor in Fig. 1-10 produces a torque T, then the motor in Fig. 1-11 should produce a torque equal to $2T$ because twice the magnets are producing twice the force. Clearly as diameter increases, there is more and more room for magnets around the rotor. So it makes sense that the ability to produce force increases linearly with diameter. Combining these two contributing factors leads to the desired relationship (1.10) that torque is proportional to diameter squared.

1.9 Units

Unless specifically noted otherwise, this text utilizes the International System of Units (SI units). Doing so eliminates the need for conversion factors that often complicate expressions and derivations. On the other hand, SI units are not universally used in practice. Several other systems of units are commonly used, each with its own advantages and disadvantages. It is assumed that the reader can convert the expressions and quantities in this text to the system of units of their choice.

1.10 Summary

This chapter developed the basic concepts involved in brushless PM motor design. Both radial flux and axial flux shapes were described. The relationship between torque and force was developed, and basic properties of magnets were used to intuitively describe how a motor works. The basic concepts of coils, windings, phases, poles, slots, teeth, and yokes were introduced. The commonly held D^2L sizing relationship was also justified intuitively. The purpose of the remaining chapters is to use and expand the intuition gained in this chapter to develop quantitative expressions describing motor operation and performance.