8 Windings

Brushless permanent magnet motors can have any even number of magnet poles N_m and any number of slots N_s. From this infinite set, only a small number of magnet pole and slot count combinations maximize use of the stator slots and lead to efficient torque production. This chapter develops the concepts required to identify these valid pole and slot count combinations for three phase motors. In addition, this chapter presents procedures for determining winding layouts for valid pole and slot combinations.

8.1 Assumptions

Since there are an infinite number of possibilities for pole and slot count combinations and for winding layouts, assumptions are required to focus or limit the scope so that desirable windings can be found. The assumptions considered here are:

(a) The motor has three phases. (Modification of the material in this chapter for other phase counts follows in a straightforward fashion.)

(b) All slots are filled. Therefore the number of slots is a multiple of the number of phases, *i.e.*, $N_s = qN_{ph}$ for some integer q. So for three phase motors, the number of slots is always a multiple of three.

(c) There are one or two coil sides in each slot. That is, the winding can be classified as a single layer or double layer winding respectively.

(d) Only balanced windings are considered. In other words, only pole and slot count combinations that result in the back EMF of phases B and C being 120 °E offset from the back EMF of phase A are considered.

(e) The number of slots per pole per phase is assumed to be less than or equal to two, where $N_{spp} = N_s/N_m/N_{ph}$ is the number of slots per pole per phase. This restriction is primarily for convenience. Most motors fulfill this requirement. If N_{spp} is greater than two, another degree of freedom is introduced that can complicate the winding layout as well as increase manufacturing cost, but seldom improve motor performance. In practice, $Nspp > 2$ often appears when a stator lamination is reused for a motor having significantly fewer magnet poles.

(f) All coils have the same number of turns and all span the same number of slots. This implies that all coils are the same size and therefore have the same resistance and inductance.

Abiding by the above list of assumptions routinely leads to motors that are capable of high performance. Moreover, these assumptions lead to motors that are readily wound. Motors can be wound that violate one or more of these assumptions; however, they may be more difficult to wind or may offer reduced performance.

8.2 Coil Span

As described in Chapter 4, coil span or coil pitch is the circumferential width of a coil. Coil span can be specified in terms of mechanical or electrical measures. In slotted motors, it is convenient to describe the coil span in terms of slots. For example, if a coil goes from slot k to slot $k + 2$, the coil span is 2 slots.

Generally speaking, the coil span for a coil should be as close to 180 °E as possible but seldom exceed it. Doing so maximizes the flux linked to the coil and therefore maximizes the back EMF induced in the coil. The exception to this rule occurs when the slot pitch exceeds 180 °E. This situation commonly occurs when the number of slots N_s is less than the number of magnet poles N_m. When the slot pitch exceeds 180 °E, the coil pitch is set to the minimum of one.

The nominal coil span as described above, can be found by defining the number of slots per magnet pole as

$$N_{sm} = \frac{N_s}{N_m} \tag{8.1}$$

This value gives the number of slots per 180 °E. As a result, the nominal coil span in slots is the integer portion of (8.1), or

$$S^* = \max\left(\operatorname{fix}\left(\frac{N_s}{N_m}\right), 1\right) \tag{8.2}$$

where the function max(·,·) returns the maximum of its two arguments and the function fix(·) returns the integer portion of its argument. The function max(·,·) is included in (8.2) to insure that the span is at least one slot when $N_s < N_m$.

Occasionally, the winding span differs from the nominal span given in (8.2). When it does, the span chosen most often is equal to $S^* - 1$. Decreasing the span decreases the length of the end turns and changes the amplitude and harmonic content of the flux linkage and resulting back EMF. In this case, the winding is said to be short pitched or chorded.

8.3 Valid Pole and Slot Counts for Double Layer Windings

Only certain combinations of magnet poles and stator slots fit the preceding winding assumptions. For example, for three phase motors the number of slots must be a multiple of three, or not all slots will be filled with two coil sides. Before considering the details of laying out a winding, it is beneficial to identify the subset of magnet pole and slot count combinations that lead to valid double layer windings.

For three phase motors, each of the three phase windings must produce a back EMF of the same amplitude and shape. More important here is that each back EMF be shifted in phase by 120 °E from the other two phases. When these three criteria on the amplitude, shape, and relative phase are met, the winding is said to be balanced.

The amplitude and shape of the phase back EMFs will be identical if the coils in each phase have the same number of turns and the same coil span and are distributed in the same way around the stator. Since these criteria are met by the winding assumptions, valid pole and slot counts are then determined by the ability to produce the 120 °E relative phase offset among the three phase windings.

With reference to **Fig. 8-1**, if the first coil of phase A uses slot 0 and slot *S*, where *S* is the chosen coil span, then the first coil of phase B must use a slot *k* and $k + S$, where *k* is chosen so that slots 0 and *k* are separated by 120 °E. The slots may also be $(120 + q360)$°E apart where *q* is any integer. That is, the principle angle between slots 0 and *k* must be 120 °E. If no such slot can be found, the chosen pole and slot count combination does not support a balanced winding.

When a slot *k* is found, each coil in phase B is shifted by *k* slots with respect to the corresponding coil in phase A. This span of $K_o = k$ slots is called the phase offset. For each coil in phase A, each corresponding coil in phase B is shifted K_o slots, thereby assuring that the individual coil back EMFs of phase B are shifted 120 °E relative to those of phase A.

Since the phase offset of K_o slots leads to a 120 °E offset between phases A and B, shifting the coils in phase C by K_o slots from those of phase B produces another 120 °E offset, thereby creating a balanced winding.

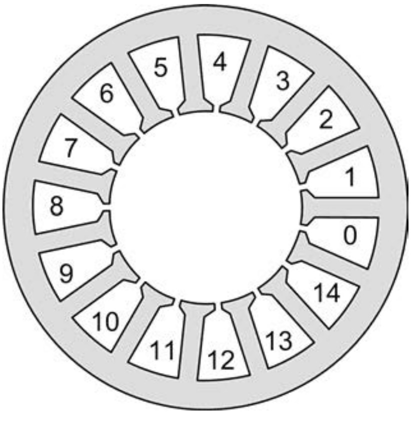

Figure 8-1. A motor having 15 slots.

Mathematically, the phase offset can be determined by identifying the angle of each slot relative to slot 0. Since the angular slot pitch is $\theta_{sp} = 360/N_s$ °M, the angle of the $(k + 1)^{\text{th}}$ slot relative to the first slot is

$$\theta_{sl}(k) = k\frac{N_m}{2}\frac{360\,°\text{E}}{N_s} = k\frac{N_m}{N_s}180\,°\text{E} \quad \text{for} \quad k = 1, 2, \cdots, N_s - 1 \tag{8.3}$$

The principle angle associated with each of these angles can be determined by using the remainder function, rem(x,y), which returns the remainder of the division x/y, (*e.g.*, rem(5,2) = 1 and rem(12,3) = 0),

$$\theta_{sl}(k) = \text{rem}\left(k\frac{N_m}{N_s}180\,°\text{E}, 360\,°\text{E}\right) \tag{8.4}$$

If it exists, the phase offset K_o is the value of k for which (8.4) equals 120 °E. It is possible that there are multiple solutions. In this case, any solution usually works equally well, so the smallest is usually chosen.

For convenience, the arguments in (8.4) can be divided by 120 °E, giving the phase offset K_o as the smallest value for which the following statement is true.

$$\text{rem}\left(\frac{3N_m}{2N_s}K_o, 3\right) = 1 \tag{8.5}$$

Because of the way the remainder function is defined, it is not possible to write a closed form solution for the phase offset. However, if a balanced winding exists, it is a simple iterative process to find it.

An alternative to the above expression can be stated that avoids use of the rem function by equating (8.3) to $(120 + q360)$ °E where q is any integer. Doing so and simplifying the result leads to the phase offset expression

$$K_o = \frac{2N_s}{3N_m}(1 + 3q) \tag{8.6}$$

where K_o is a valid phase offset if evaluation of (8.6) for an integer value of q in the range 0 to $N_m/2 - 1$ produces an integer result.

As an example, consider the four pole, twelve slot case considered in Chapter 4. Iterating (8.5) shows that $K_o = 2$. Therefore if phase A starts in slot 0, phase B starts in slot 2 and phase C starts in slot 4. This agrees with the coil placement shown in Fig. 4-12. For the four pole, fifteen slot case considered in Chapter 4, $K_o = 10$. Thus, if phase A starts in slot 0, phase B starts in slot 10, and phase C starts in slot 20. In this case, slot 20 is the slot labeled rem(20,15) = 5.

8.4 Winding Layout for Double Layer Windings

Because there is greater flexibility in the choice of rotor poles N_m and stator slots N_s in brushless permanent magnet motors, there is greater flexibility in the placement of windings. For this reason, generic winding terms applied to other motor types such as lap, wave, concentric, and sinusoidally distributed have less meaning. Similarly, terms such as distribution factor, pitch factor, and winding factor that describe the effect a winding layout has on the shape of the flux linkage and resulting back EMF have less significance. This is particularly true because direct computation of the harmonic content of the back EMF is so easily accomplished with a computer.

The winding layout developed here leads to a double layer lap winding that appears in most brushless permanent magnet motors. The layout is both manufacturable and maximizes motor performance. While the winding layout developed here can be modified to produce a wave winding that may or may not be single layer, doing so does not generally improve performance. According to the *BLv* and *BLi* laws, the distribution of the coil end turns does not influence back EMF or torque; rather, it is the slot placement of coils that influences back EMF and torque. The end turns exist solely to transport current from one slot to the next. In other words, the *BLv* and *BLi* laws don't say anything about the end turns, so how the end turns are laid out simply does not play a role in back EMF or torque production. However, end turn layout does influence coil resistance, inductance, and manufacturability.

The goal in laying out a winding is to place coils having a span of S in slot pairs such that relative angular coil midpoints are as close to 0 °E and 180 °E separation as possible. Coils close to 0 °E are wound in one direction and coils close to 180 °E are wound in the reverse or opposite direction since the magnet flux is in the opposite direction at 180 °E. For example, consider the integral slot pitch, four pole, twelve slot motor shown in **Fig. 8-2**. (Note that the slots in this figure are numbered starting with the number one. This is different but otherwise equivalent to numbering that starts with the number zero as shown in Fig. 8-1.) In Fig. 8-2, coils having midpoints at θ_1 and θ_3 are at the same angle designated 0 °E and are wound in one direction. On the other hand, coils having midpoints at θ_2 and θ_4 are 180 °E away from θ_1 and θ_3 respectively and are wound in the opposite direction. To signify the relative coil direction, the terms *In* and *Out* are used as shown in Fig. 8-2. *In* refers to the coil side entering a slot and *Out* refers to a coil side coming out of a slot.

In fractional slot motors, it is not possible to align all coils at 0 °E or 180 °E separation. As a result, coil locations must be chosen that are as close as possible to 0 °E and 180 °E separation. For those coils closest to 180 °E, the reverse or opposite winding direction is used. This effectively shifts the coil angle by 180 °E back toward 0 °E. The required number of coils per phase are then chosen from this list such that the winding assumptions stated earlier are met.

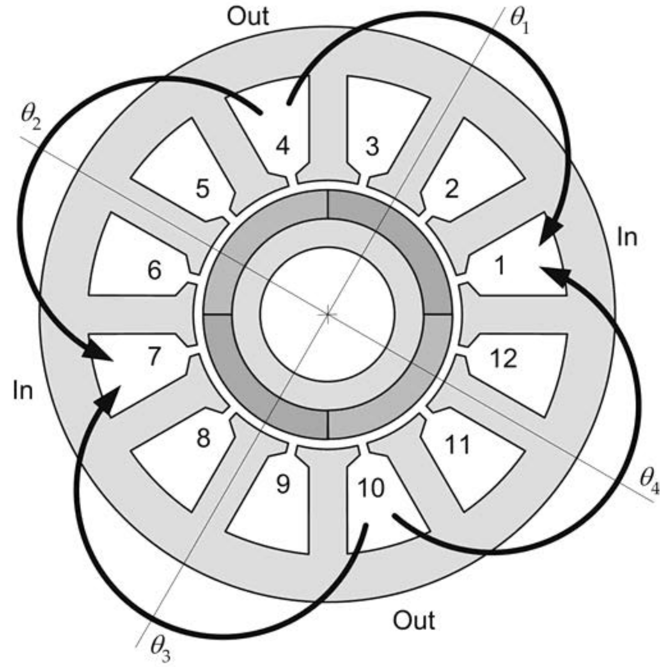

Figure 8-2. An integral slot pitch, four pole, twelve slot motor.

As stated in assumption (b), the number of slots is always a multiple of three for three phase motors. Since each coil in a double layer winding fills two slots one half full, each coil effectively fills one net slot. As a result, the number of coils per phase is

$$N_{cph} = \frac{N_s}{N_{ph}} = \frac{N_s}{3} \tag{8.7}$$

This expression gives the number of coil locations that must be found for each phase. The coil locations for other phases are found by applying the phase offset K_o twice to the coil locations found for phase A.

Example

To illustrate how coil locations are found, consider the four pole, fifteen slot motor shown in **Fig. 8-3**. Based on the nominal coil span of three, *i.e.*, $S^* = 3$, if the coil going in slot 1 and out slot 4 is at 0 °E, then a coil wound in the same direction in slots 2 and 5 is at a offset angle equal to one slot pitch, or $\theta = \theta_s = (N_m/N_s)\cdot 180$ °E or 48 °E. Similarly, a coil wound in the same direction in slots 5 and 8, is at a offset angle of $\theta = 4\theta_s = 4\cdot 48$ °E or 192 °E. If this latter coil is wound in the opposite direction as that shown in the figure, *i.e.*, the *In* slot becomes slot 8 and the *Out* slot becomes slot 5, then the offset angle of this coil becomes 192 °E – 180 °E = 12 °E.

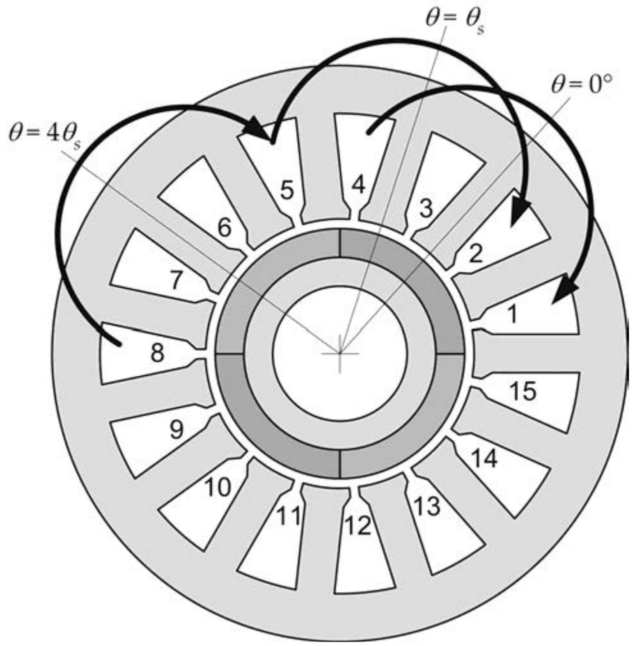

Figure 8-3. A four pole, fifteen slot motor.

In other words, the offset angle of all potential coils having an *In* slot of *k* is

$$\theta_c(k) = (k-1)\frac{N_m}{N_s}180\,°\text{E} \tag{8.8}$$

For the four pole, fifteen slot motor in Fig. 8-3, these angles and associated *In* and *Out* slots are

Coil	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Angle	0	48	96	144	192	240	288	336	384	432	480	528	576	624	672

In	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Out	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3

These angles are correct but difficult to decipher because they extend outside the range -180 °E $\le \theta \le$ 180 °E. Mathematically this problem can be corrected by finding the principle angle within this range by applying the rem function expression

$$\theta = \text{rem}\left(\theta + 180°, 360°\right) - 180° \tag{8.9}$$

Doing so, the above coil angles become

Coil	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Angle	0	48	96	144	-168	-120	-72	-24	24	72	120	168	-144	-96	-48

In	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Out	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3

For coil angles having a magnitude greater than 90 °E, the coil direction is reversed, thereby changing the coil angle by 180 °E. Performing this operation for the four pole, fifteen slot motor being considered modifies the above coil data to

Coil	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Angle	0	48	-84	-36	12	60	-72	-24	24	72	-60	-12	-36	84	-48

In	1	2	6	7	8	9	7	8	9	10	14	15	1	2	15
Out	4	5	3	4	5	6	10	11	12	13	11	12	13	14	3

Given this table of all potential coils for phase A, choosing those closest to 0 °E and minimizing the total spread of angles will generally maximize motor performance. Since there are five coils per phase for this motor, coils numbered 1, 5, 8, 9, and 12 are closest to 0 °E and have a total spread of 24 °E – (-24) °E = 48 °E. Selecting these coils from the above data and sorting them by magnitude gives

Coil	1	5	12	9	8
Angle	0	12	-12	24	-24

In	1	8	15	9	8
Out	4	5	12	12	11

To confirm that this choice of coils satisfies all the winding assumptions, these coils and their associated phase B and phase C counterparts are shown in the Table 8-1. Here the windings are tabulated by slot number, and the coil offset of $K_o = 10$ slots has been used to place the corresponding coils of phases B and C. Since each row in the table has two entries, each slot is full and contains two coil sides. Therefore, the above table identifies a valid winding.