9 Electrical Control

Given the back EMF of each phase, the production of torque requires electrical control of the currents in each motor phase winding. In this chapter, torque production in a brushless permanent magnet motor is studied without introducing detailed power electronic circuitry. While such circuitry is inherently required, its introduction can easily cloud the underlying fundamental requirements of the motor. Therefore, this chapter focuses on what the motor requires rather than on what the power electronics can produce.

9.1 Fundamentals of Torque Production

Mutual torque production is best understood through the use of conservation of energy concepts as demonstrated earlier in Chapter 3 and by (3.43). Each motor phase winding is composed of a resistive component, an inductive component, and a back EMF as shown in **Fig. 9-1**. Application of voltage v across the winding causes a current i to flow through the winding. The current flow creates ohmic losses or heat in the resistor R_{ph} and creates a magnetic field that stores energy in the inductance L_{ph}. When the phase current flows through the back EMF source e_{ph}, the source absorbs instantaneous power equal to the product $e_{ph} i$. This power must go somewhere. It does not create heat like the phase resistance; it does not store energy in a magnetic field like the phase inductance. To satisfy conservation of energy, this power is converted to mechanical power, which is given by the product of torque and speed $T\omega$ as given by (3.33), *i.e.*, $e_{ph} i = T\omega$, where ω is in radM/s.

When a motor has more than one phase winding, conservation of energy must apply simultaneously for all phases. For three phase motors being considered here, this implies that

$$T\omega = e_a i_a + e_b i_b + e_c i_c \tag{9.1}$$

where e_x and i_x for $x = a, b, c$ are the respective back EMFs and currents in the three motor phases A, B, and C. Recognizing that the amplitudes of back EMFs are lin-

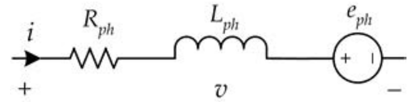

Figure 9-1. Electrical circuit model for one phase winding.

early proportional to speed ω, the back EMFs in (9.1) can be written as $e_x = k_x\omega$ where k_x is the speed-independent shape of the back EMF having units of V/radM/s. Substituting this relationship into (9.1) gives

$$T(\theta) = k_a(\theta)i_a(\theta) + k_b(\theta)i_b(\theta) + k_c(\theta)i_c(\theta) \tag{9.2}$$

where the position dependence of the torque, back EMF shapes, and currents has been shown explicitly.

Based on (9.2), it is clear that the speed-independent back EMF shapes $k_x(\theta)$ for $x = a, b, c$ are also torque constants because they describe a proportionality between torque and current. This fact is also apparent because the units of $k_x(\theta)$, V/radM/s are identical to N·m/A, which is torque divided by current.

In three phase motors with balanced windings, the back EMFs and currents of the three phases have the same shape but are offset from each other by $\theta_{phe} = 2\pi/3$ radE or 120 °E from each other. Using this angular relationship, (9.2) can be rewritten as

$$T(\theta_e) = k_a(\theta_e)i_a(\theta_e) + k_a(\theta_e - \theta_{phe})i_a(\theta_e - \theta_{phe}) + k_a(\theta_e + \theta_{phe})i_a(\theta_e + \theta_{phe}) \tag{9.3}$$

where angles are now expressed in electrical measure since the back EMFs and currents are periodic with respect to electrical measure. That is, the back EMFs and currents repeat every 360 °E. In addition, the fact that -240 °E = +120 °E allows $\theta_e - 2\theta_{phe}$ to be written as $\theta_e + \theta_{phe}$. From (9.3), it is clear that given the motor back EMF shape $k_a(\theta_e)$, torque production is determined solely by specifying the current shape in phase A, $i_a(\theta_e)$.

In practice, the desired torque is almost always a constant proportional to the amplitude of the current. That is, the motor should produce torque that does not vary as a function of position θ but does vary linearly with current amplitude. By following these desired characteristics, the motor becomes an easily-controlled source of torque that promotes optimum control of the load attached to the motor. The existence of position dependent variations in the mutual torque, *i.e.*, torque ripple, causes the motor load to repeatedly accelerate when the torque increases and decelerate when the torque decreases. This repeated acceleration and deceleration is smoothed significantly by the motor and load inertias, but it can affect the performance of the motor when used in positioning applications when velocity approaches zero at position endpoints.

Given this description of the desired motor torque, the goal in choosing the phase A current shape $i_a(\theta_e)$ is to produce constant torque. In practice, achieving constant torque has been accomplished by considering two "standard" back EMF shapes—trapezoidal and sinusoidal. When the motor has a trapezoidal back EMF, the motor is often referred to as a brushless DC motor, whereas when the back EMF is sinusoidal, the motor is often referred to as a permanent magnet synchronous motor or more simply as a sinewave motor. There is nothing magical about these terms. The fundamentals of torque production are identical, although the mathematics used to describe the two motors may be dramatically different. Historically, the two motors originated from different application areas. The term brushless DC motor comes from the fact that a brushless DC motor approximates the operation of a permanent magnet brush DC motor with power electronics taking the place of the brushes and commutator. The term permanent magnet synchronous motor describes an AC synchronous motor whose field excitation is provided by permanent magnets rather than by field coil that is energized through brushes. Because of this difference in origin, the two motors are often mistakenly assumed to be much different from one another. What is different is the mathematics customarily used to describe them. The brushless DC motor is described in terms such as a torque constant and back EMF constant, whereas the permanent magnet synchronous motor is described in terms such as a rotating air gap MMF, synchronous reactance, and vector control using a coordinate system based on direct and quadrature axes.

In the sections that follow, both of these motor types will be discussed as well as more general motor electronic drive configurations. The fundamentals of vector control and common drive topologies are illustrated in the next chapter.

9.2 Brushless DC Motor Drive

Ideal Torque Production

As stated earlier, a brushless DC motor generally describes a motor having a trapezoidal back EMF. For this case, the phase currents are rectangular pulses, sometimes loosely called squarewave currents. While (9.3) can be used to describe torque production for this motor, it is easier to understand this configuration graphically as shown in **Fig. 9-2**, where the three phases have been labeled A, B, and C respectively.

In the figure, the back EMF shapes, *i.e.*, the back EMFs divided by speed, are trapezoids having 2/3 duty cycle. That is, for each 180 °E the back EMF shape is constant over 120 °E. The current associated with each back EMF is composed of rectangular pulses having a 2/3 duty cycle, where the nonzero portions of the pulses are aligned with the flat areas of the respective back EMF shapes and the polarity of the current matches that of the back EMF. Following (9.2), the constant torque produced is shown at the bottom of the figure. Over each 60 °E segment, positive current flows in one phase, current flows out of another, and no current flows in the third phase. The letters below the constant torque line signify the two phases carrying current, with the overbar denoting negative current flow or current flow out of a phase. During every 60 °E where the back EMF of a phase makes a transition, the current in one phase remains unchanged, while the current in another goes to zero, and the current in the third becomes nonzero. Over 360 °E, there are six transitions or commutations before the sequence repeats. As a result, this motor drive is often called a six step drive.

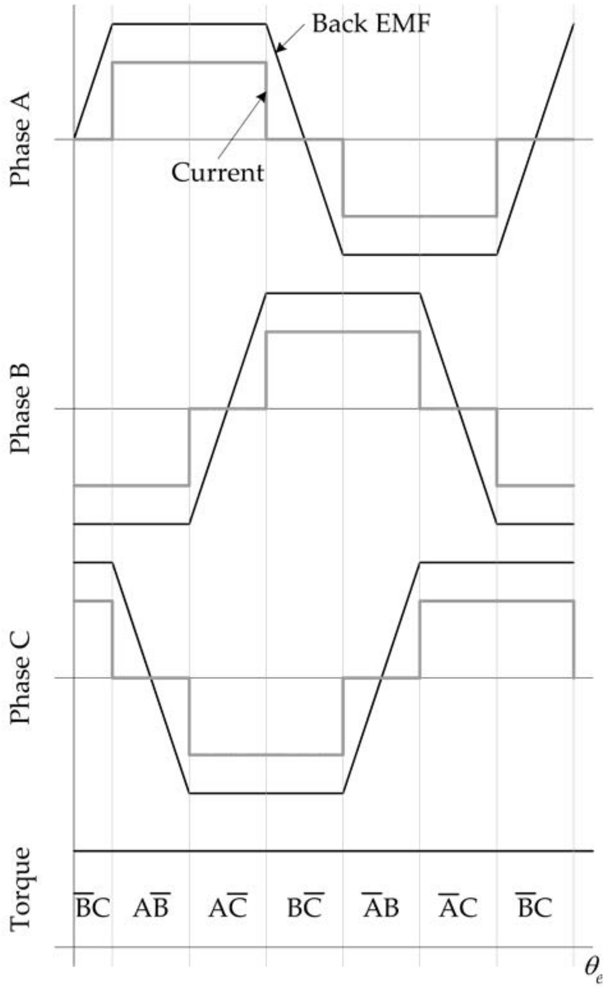

Figure 9-2. Brushless DC motor drive waveforms.

If the amplitude of the back EMF shapes, *i.e.*, the back EMFs divided by speed ω in radM, in Fig. 9-2 is K_p and the amplitude of the current is I_p, then the torque produced by the brushless DC motor drive configuration is

$$T(\theta_e) = 2K_pI_p \tag{9.4}$$

which shows that the mutual torque is constant and proportional to the current amplitude. The factor of two in (9.4) appears because two phases produce a torque equal to K_pI_p at all instants. In this case, $2K_p$ can be thought of as the torque constant since it is the factor relating peak phase current to total motor torque.

The fundamental appeal of this brushless DC motor drive configuration is that position feedback need only identify the commutation points every 60 °E. As a result, three simple Hall effect devices can be used to identify the commutation points. If the Hall effect devices are properly aligned with the back EMFs, processed Hall effect device signals can easily produce logic signals whose transitions occur at the desired commutation points.

Motor Constant

Given the torque produced in (9.4), the motor constant K_m for the brushless DC motor configuration can be found by computing the I^2R losses incurred. The RMS value of the ideal rectangular pulse currents shown in Fig. 9-2 is given by

$$I_{rms} = \sqrt{\frac{2}{3}} I_p \tag{9.5}$$

This RMS current flows through all three phase resistances producing total I^2R losses of

$$P = 3 I_{rms}^2 R_{ph} \tag{9.6}$$

Substituting (9.4), (9.5), and (9.6) into the motor constant expression (4.46) gives

$$K_m = K_p \sqrt{\frac{2}{R_{ph}}} \tag{9.7}$$

This expression shows that the motor constant is directly proportional to the amplitude of the back EMF shape K_p, back EMF constant, or one half the torque constant $2K_p$. Since this analysis applies to the ideal case that cannot be achieved in practice, (9.7) represents the maximum achievable motor constant. Real brushless DC motor drive configurations will exhibit a somewhat lower motor constant.

Torque Ripple

The biggest disadvantage of the brushless DC motor drive configuration is the physical inability to generate the ideal rectangular pulse currents. As shown in the figure, the currents must make the required transitions instantaneously, *i.e.*, di/dt or $\omega_e(di/d\theta_e)$ is infinite at the transitions. In reality, the transitions require finite time. As a result, torque ripple, called commutation torque ripple, is created at each commutation point during the finite transition time of each phase current.

In addition to significant commutation torque ripple, the brushless DC motor drive configuration produces torque ripple whenever the back EMF or current shapes deviate from their ideal characteristics shown in Fig. 9-2. For example, if the back EMF or current shapes do not have a uniform amplitude from phase to phase or are not flat over the desired 120 °E intervals when torque is produced, torque ripple appears.

Because torque ripple is difficult to eliminate in the brushless DC motor drive configuration, it is seldom used in applications where minimum torque ripple is required. However, in velocity applications such as fans and pumps where motor speed and inertia are sufficiently high, torque ripple has little effect because of the inherent filtering provided by the inertia.

Line-to-Line Back EMF

Finally, it is important to note that the trapezoidal back EMFs shown in Fig. 9-2 are the back EMFs as measured across individual phase windings. In the common situation where the motor windings are connected in the Y-connection where one end of each phase winding is connected together inside the motor, the back EMF measured across two of the external terminals is the difference between two of the phase back EMFs. For example, **Fig. 9-3** illustrates the back EMF that appears between phases A and B when the phase back EMFs are ideal trapezoids. The line-to-line back EMF that appears between phases A and B remains trapezoidal, but has larger transition regions between its positive and negative extremes. The resulting amplitude is twice that of the phase back EMFs. In real motors, the phase back EMFs are trapezoids with smoothed or rounded transitions and the resulting the line-to-line back EMFs are also smoothed. When there is sufficient smoothing, the line-to-line back EMFs more closely resemble a sinusoidal shape.

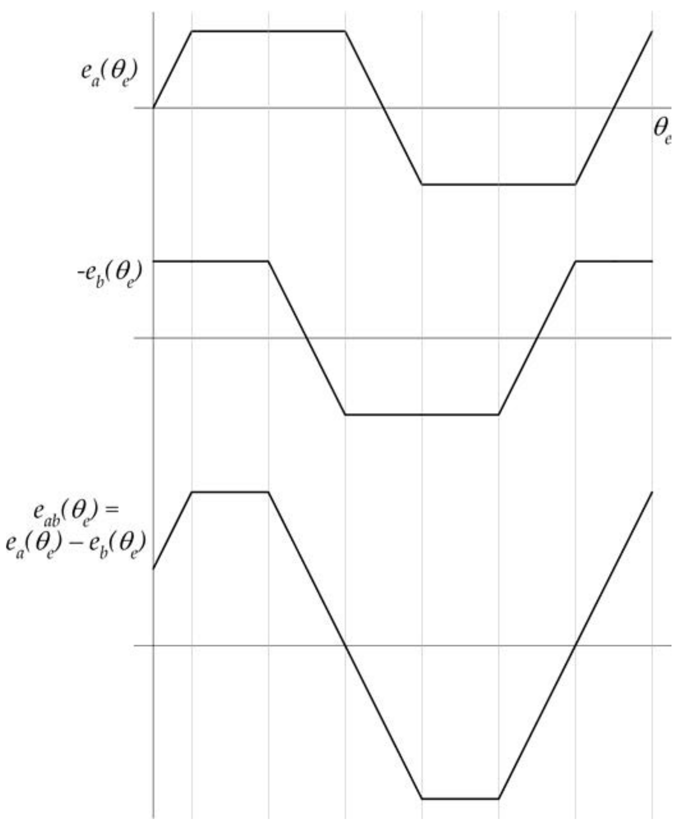

Figure 9-3. Line to line back EMF from phase back EMFs.

As an alternative to Fig. 9-2, the relationship between line-to-line back EMFs and individual phase currents required to produce constant torque are shown in **Fig. 9-4.** Here, the phase currents exist over the 60 °E flat topped regions of the respective line-to-line back EMF and continue during the next 60 °E of the back EMF transition. For example, in the upper part of the figure, $i_a(\theta_e)$ is positive over the 60 °E positive flat top region of $e_{ab}(\theta_e)$ and continues positive for an additional 60 °E before going to zero for the next 60 °E. Later, $i_a(\theta_e)$ is negative over the 60 °E negative flat top region of $e_{ab}(\theta_e)$ and continues negative for an additional 60 °E before going to zero for the next 60 °E. Similar statements can be made for the other two phases as shown in the figure.

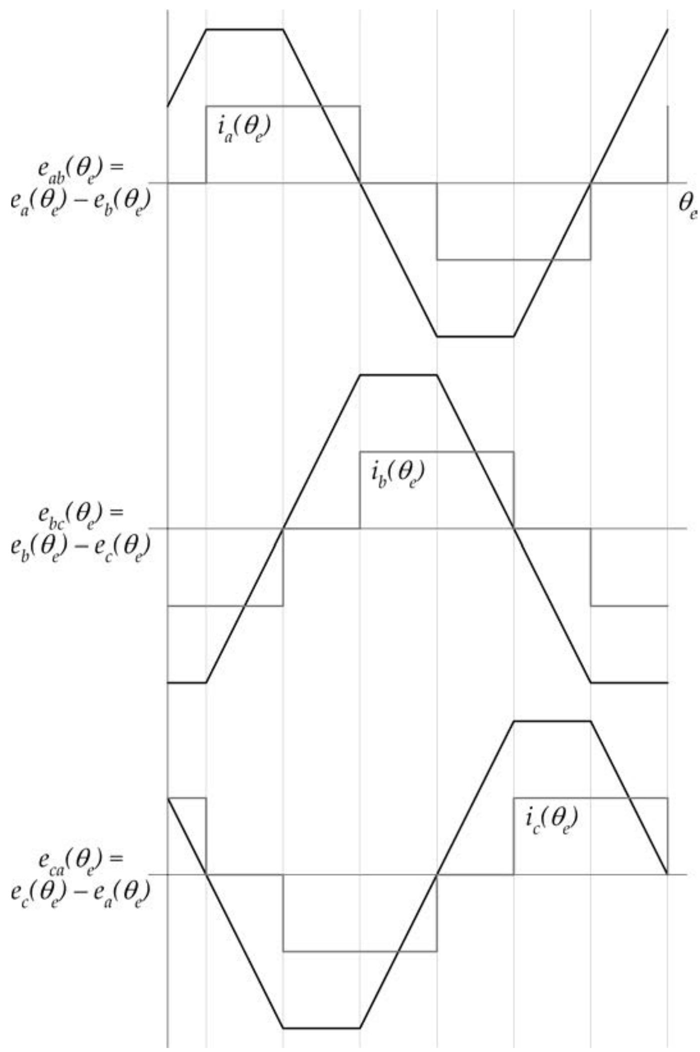

Figure 9-4. Line to line back EMFs ans associated phase currents.