10 Vector and Field Oriented Concepts

The implementation of motor control algorithms has become dominated by digital implementations in which mathematical operations on digitized signals have replaced analog circuitry operating on continuous voltage and current signals. As a result, mathematically based control algorithms that are difficult, if not impossible, to implement with analog circuitry are simpler to implement than traditional analog control algorithms. For brushless permanent magnet motors, the most common digital control algorithms are based on a coordinate transformation introduced by R. H. Park, which is now nearly a century old. Various aspects of this transformation and its use have been denoted by a variety of titles including, Park transformation, Clark transformation, two axis theory, dq_0 transformation, qd_0 transformation, vector control, field orientation, reference frame theory, and space vectors. Because of the prevalence of control algorithms based on the work of Park, this chapter introduces the concepts involved. This chapter focuses on the fundamental geometric concepts that aid in understanding what often appears to be unjustified and confusing mathematical statements and derivations in other works. Once the fundamental concepts are understood from a geometric perspective, the strictly mathematical derivations found elsewhere make more sense.

10.1 Introduction

The term vector control denotes motor control based on a description of variables in terms of vectors. The term *vector* refers to describing magnetic fields and currents that produce them in terms of vectors in spatial coordinate systems, *e.g.*, polar coordinates. One of these spatial coordinate systems is associated with or fixed to the rotor and the other is associated with the stator. Depending on your frame of reference, *i.e.*, reference frame, the rotor either rotates relative to a fixed stator, or equivalently the stator rotates relative to a fixed rotor. Since the rotor is concentric within the stator, the two coordinate systems share the same origin and angular displacement alone describes the relationship between the two coordinate systems.

In the sections that follow these coordinate system concepts will be used to develop the fundamental ideas behind vector and field oriented analysis and control of brushless permanent magnet motors. The ideas presented naturally involve motor phase currents and the magnetic fields they produce. However, the derived transformations apply equally to other motor parameters as well, including back EMF, phase voltages, *etc.*

10.2 Two Phase Motors and the Park Transformation

Consider the simple two phase motor shown in **Fig. 10-1**. For visual simplicity, $N_m = 2$ was chosen and the stator is shown with four poles, with one opposing set of poles, A and $\bar{A}$, wound as phase A, the other pair wound as phase B, and the phases spaced $\theta_{phe} = 90$ °E = 90 °M apart. A two phase motor was chosen because it most easily facilitates the visual geometric development of vector concepts. Later, three phase motors will be analyzed in the same way, leading to an equivalent description.

Figure 10-1 depicts two Cartesian coordinate systems. The rotor coordinate system has axes labeled d and q, that identify the direct and quadrature axes respectively. Direct denotes the axis directly in line with the North pole of the rotor magnetic field. Quadrature denotes the axis perpendicular to the direct axis. The stator coordinate system has perpendicular axes labeled α and β, with the α-axis

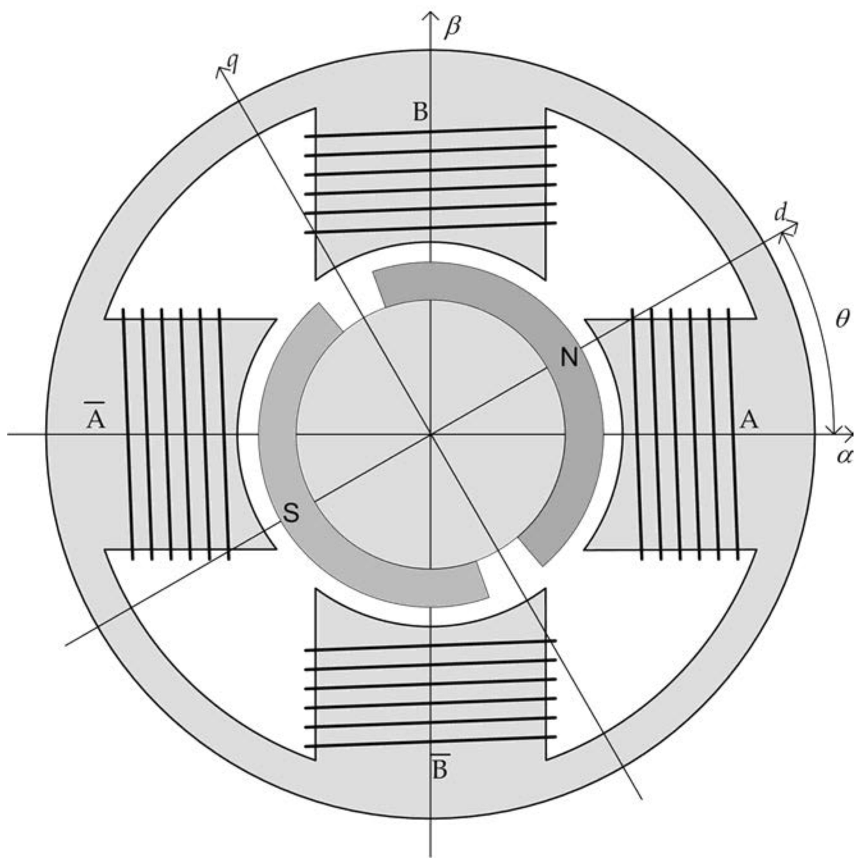

Figure 10.1 A conceptual two phase motor.

aligned with phase A and the β-axis aligned with phase B. The angular offset between the rotor and stator axes is denoted θ as shown in the figure.

The angular offset between the rotor and stator axes dictates how the phases must be commutated to produce torque. For example, when θ is equal to zero as shown in **Fig. 10-2***a*, the rotor permanent magnet flux is directed through the coils of phase A. At this position, the flux linked to phase A is a maximum and the flux linked to the phase B coils is zero since the net magnetic field crossing the air gap at the phase B poles is zero. If current is directed through the phase A coils so that the electromagnet poles created at the air gaps of phase A are as shown in Fig. 10-2*a*, no torque is produced. Only radial forces are created. Therefore, when θ is equal to zero, current in phase A does not produce torque, and so it should be zero. However, torque is produced when current is directed through the phase B coils so that the electromagnetic poles created at the air gaps of phase B are as shown in Fig. 10-2*a*. In this case, the phase B poles at the air gap produce both

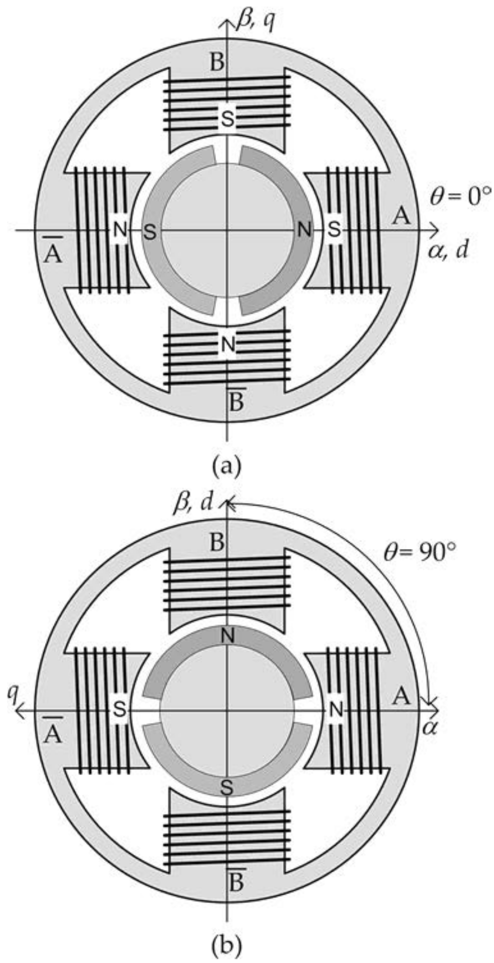

Figure 10-2. A conceptual two phase motor with the rotor position at (a) 0°, and (b) 90°.

repulsion torque pushing the South rotor pole away and attraction torque pulling the North rotor pole closer. Torque is produced when the stator current and its associated stator magnetic field leads the rotor magnetic field by 90 °E. Alternatively, torque is produced when the stator current and its associated magnetic field are oriented along the rotor quadrature axis.

When θ is equal to 90 °E as shown in Fig. 10-2*b*, the rotor permanent magnet flux linking the coils of phase B is maximized and phase B current produces no torque. Here, the flux linking the phase A coils is zero and only phase A current produces torque as indicated by North and South electromagnet poles on phase A in the figure. Once again, the torque producing stator current is along the rotor quadrature axis.

In conventional synchronous motor terminology, the 90 °E phase lead between the stator current and the rotor magnetic field is known as the optimum torque angle. In synchronous motors driven without position feedback, *e.g.*, step motors, or brushless permanent magnet motors driven without position feedback, the torque angle varies dynamically between 0 and 90 °E in response to the load torque. When the load torque is small, the torque angle is near 0 °E. When the load torque equals the motor torque capacity, the torque angle is 90 °E. When the load torque exceeds the torque capacity, the torque angle goes past 90 °E, the motor torque decreases, and the motor stalls. In common brushless permanent magnet motors, the torque angle is ideally kept at 90 °E by position feedback and the phase current amplitudes are dynamically adjusted to meet instantaneous load torque requirements. Doing so maximizes energy conversion efficiency.

Space Vectors

When the rotor quadrature axis is not aligned with either the α or β stator axis, neither stator current acting alone produces current at the optimum torque angle of $\theta + 90$ °E. In this case, vector addition of the stator currents gives the direction of the net stator current and resulting magnetic field. Since these current vectors are defined in terms of spatial angles, they are often called space vectors as shown in **Fig. 10-3**.

In Fig. 10-3, $i_\alpha(\theta)$ and $i_\beta(\theta)$ are the phase A and B currents respectively. The amplitude or length of these vector components vary instantaneously to meet the torque required as directed by the motor drive electronics. However, their spatial angles are fixed along the α- and β-axes by their physical placement within the motor.

In many texts, currents are written as functions of time, *e.g.*, $i_\alpha(t)$ and $i_\beta(t)$ rather than functions of angular position θ. Both representations are equally valid, however this text adopts angular position notation because the currents must be controlled as a function of position to produce torque. The fact that currents vary as a function of time is of secondary importance since their shape versus angular position is what determines motor performance.

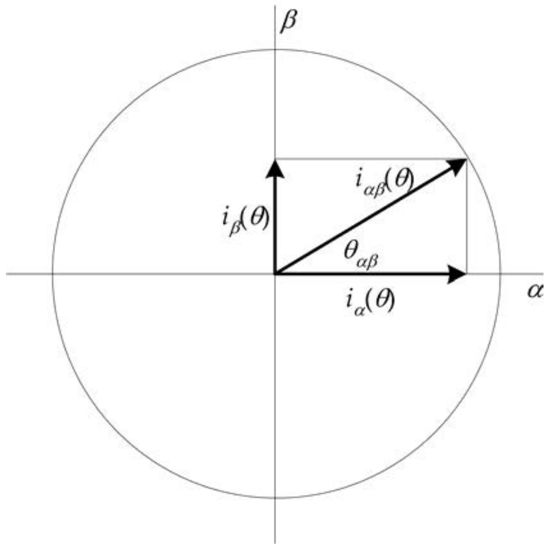

Figure 10-3. Graphical definition of space vectors in rectangular and polar forms.

As shown in Fig. 10-3, the vector sum of $i_\alpha(\theta)$ and $i_\beta(\theta)$ is given by the amplitude $i_{\alpha\beta}(\theta)$ and the angle $\theta_{\alpha\beta}$. Mathematically, these currents are described in rectangular and polar coordinates in space as

$$\vec{i}_{\alpha\beta} = i_\alpha(\theta)\vec{\alpha} + i_\beta(\theta)\vec{\beta} = i_{\alpha\beta} \measuredangle \theta_{\alpha\beta} \tag{10.1}$$

where

$$\begin{aligned} \vec{\alpha} &= \text{unit vector along } \alpha \text{ axis} \\ \vec{\beta} &= \text{unit vector along } \beta \text{ axis} \\ i_{\alpha\beta} &= \sqrt{i_\alpha^2 + i_\beta^2} = \text{space vector length} \\ \theta_{\alpha\beta} &= \arctan\left(\frac{i_\beta(\theta)}{i_\alpha(\theta)}\right) = \text{space vector angle} \end{aligned} \tag{10.2}$$

To maintain a torque angle of 90 °E, the stator current space vector angle $\theta_{\alpha\beta}$ must be equal to

$$\theta_{\alpha\beta} = \theta + 90\,°\text{E} \tag{10.3}$$

To maintain constant torque, the stator current space vector amplitude $i_{\alpha\beta}$ must be constant with respect to angle θ. Varying the torque produced is then simply a matter of varying the current space vector amplitude. These ideas can be visualized by imagining the process of dragging the tip of the current space vector in Fig. 10-3 around in a circle in a counterclockwise fashion to meet the torque angle requirement in (10.3) and varying the diameter of the circle passing through the tip of $i_{\alpha\beta}$ as needed to meet the load torque requirement.

Park Transformation

Based on Fig. 10-2 and its accompanying description, only the component of the stator current and its magnetic field that is along the rotor's quadrature axis leads to torque production. This fact was first demonstrated in the previous chapter, *e.g.* (9.21). Stator current and its associated magnetic field along the rotor's direct axis creates I^2R losses, but does not produce torque. Therefore, it is convenient to describe the stator current in terms of the rotor coordinate system. Mathematically, this is called a reference frame transformation because coordinate axes form the reference for describing the vector components as in (10.1) and Fig. 10-3. All we want to do here is describe the current space vector in terms of a different reference frame, *i.e.*, a different coordinate system. Because the stator and rotor coordinate systems share the same origin and differ in angle only, the coordinate systems are related by rotation as shown in **Fig. 10-4**.

Using the components of $i_\alpha(\theta)$ and $i_\beta(\theta)$ along the d- and q-axes respectively as shown in Fig. 10-4, the corresponding d- and q-component currents can be written as

$$\begin{aligned} i_d(\theta) &= i_\alpha(\theta)\cos(\theta) + i_\beta(\theta)\sin(\theta) \\ i_q(\theta) &= -i_\alpha(\theta)\sin(\theta) + i_\beta(\theta)\cos(\theta) \end{aligned} \tag{10.4}$$

where $i_\alpha(\theta)\cos(\theta)$ and $i_\beta(\theta)\sin(\theta)$ are the two components along the d-axis and $-i_\alpha(\theta)\sin(\theta)$ and $i_\beta(\theta)\cos(\theta)$ are the two components along the q-axis. These relationships are commonly written in vector-matrix format as

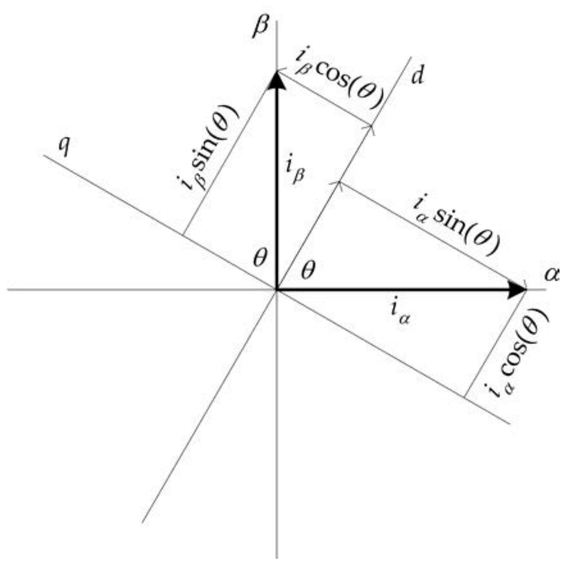

Figure 10-4. Park transformation relationship between the stator and rotor coordinate systems.

$$\begin{bmatrix} i_d(\theta) \\ i_q(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_\alpha(\theta) \\ i_\beta(\theta) \end{bmatrix} \qquad (10.5)$$

$$\boldsymbol{i}_{dq} = \mathbf{P}\, \boldsymbol{i}_{\alpha\beta}$$

This relationship is known as the Park transformation. The inverse relationship, known as the inverse Park transformation, relates the currents in the opposite direction and can be written in matrix-vector format as

$$\begin{bmatrix} i_\alpha(\theta) \\ i_\beta(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_d(\theta) \\ i_q(\theta) \end{bmatrix} \qquad (10.6)$$

$$\boldsymbol{i}_{\alpha\beta} = \mathbf{P}^{-1}\, \boldsymbol{i}_{dq}$$

The two currents $i_d(\theta)$ and $i_q(\theta)$ do not exist physically, but they do have physical meaning. The current $i_d(\theta)$ produces that portion of the stator magnetic field that points in the same direction as the North rotor permanent magnet field. As shown in Fig. 10-2*a*, it is equal to $i_\alpha(\theta)$ when $\theta = 0$ °E, and it is equal to $i_\beta(\theta)$ when $\theta = 90$ °E as shown in Fig 10-2*b*. Similarly, the current $i_q(\theta)$ produces that portion of the stator magnetic field that is in quadrature, *i.e.*, perpendicular to, the rotor permanent magnet field. The stator magnetic field is 90 °E ahead of the North rotor permanent magnet field. As shown in Fig. 10-2*a*, it is equal to $i_\beta(\theta)$ when $\theta = 0$ °E, and it is equal to $-i_\alpha(\theta)$ when $\theta = 90$ °E as shown in Fig 10-2*b*. This physical interpretation explains the meaning behind the terms field orientation and field oriented control.

The significance of the Park transformation is best illustrated by deriving the torque expression for the two phase motor in both the *α-β* and *d-q* coordinate systems. Using the same electromechanical power balance relationship as that used in (9.1) for the three phase motor, the two phase motor obeys the relationship

$$T\,\omega_m = e_\alpha(\theta)\, i_\alpha(\theta) + e_\beta(\theta)\, i_\beta(\theta) \qquad (10.7)$$

where ω_m is speed in radM/s, e_α and e_β the respective position dependent back EMFs, and i_α and i_β are the phases A and B currents respectively.

The back EMFs in (10.7) are related to the permanent magnet flux linking the respective phase windings by Faraday's law (7.43). For simplicity, it is convenient and commonly assumed that the flux linking the coils exhibits a sinusoidal shape. Using the description of Fig. 10-2, the flux linkages of the two phase windings are

$$\begin{aligned} \lambda_\alpha(\theta) &= \lambda_p \cos(\theta) \\ \lambda_\beta(\theta) &= \lambda_p \cos(\theta - 90\,°) = \lambda_p \sin(\theta) \end{aligned} \qquad (10.8)$$

where λ_p is the peak value of the flux linkage. These equations match the description of Fig. 10.2 because the flux linking the phase A coil is maximum at $\theta = 0$ °E and the flux linking the phase B coil is zero at $\theta = 0$ °E. Likewise the flux linking the phase A coil is zero at $\theta = 90$ ° and the flux linking the phase B coil is maximum at $\theta = 90$ °.

Applying Faraday's law to the flux linkages in (10.8) gives the respective back EMFs as

$$\begin{aligned} e_\alpha(\theta) &= \frac{d\theta}{dt}\frac{d\lambda_\alpha(\theta)}{d\theta} = -\omega_e \lambda_p \sin(\theta) = -\omega_m K_p \sin(\theta) \\ e_\beta(\theta) &= \frac{d\theta}{dt}\frac{d\lambda_\beta(\theta)}{d\theta} = \omega_e \lambda_p \cos(\theta) = \omega_m K_p \cos(\theta) \end{aligned} \tag{10.9}$$

where ω_e is speed in radE/s, ω_m is speed in radM/s and $K_p = (\omega_e/\omega_m)\lambda_p$ is the individual phase back EMF constant having units V/radM/s.

Assuming that the phase currents exhibit a sinusoidal shape as well, and using Fig. 10-2 and its description, the currents in the phase A and B windings are

$$\begin{aligned} i_\alpha(\theta) &= -I_p \sin(\theta) \\ i_\beta(\theta) &= I_p \cos(\theta) \end{aligned} \tag{10.10}$$

where I_p is the peak current value. These currents lead their respective flux linkages by the maximum torque angle of 90 °E or $\pi/2$ radE as shown in **Fig. 10-5**. Substituting (10.9) and (10.10) into (10.7) and simplifying gives the motor mutual torque as

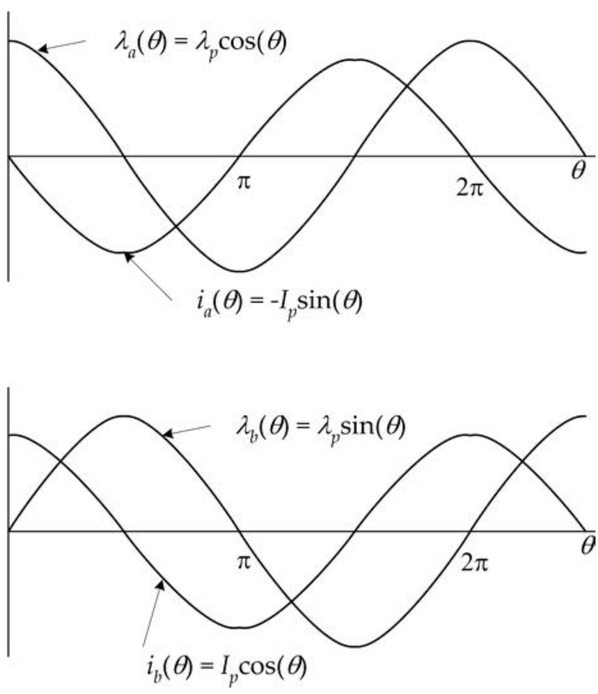

Figure 10-5. Relationship between currents and flux linkages.

$$T = K_p I_p \sin^2(\theta) + K_p I_p \cos^2(\theta) = K_p I_p \tag{10.11}$$

This expression describes the ideal and constant torque for a two phase motor having sinusoidal back EMFs and sinusoidal currents. The corresponding three phase equivalent expression is given by (9.9). Comparing the two expressions, each phase in each motor contributes $K_p I_p/2$ to the total torque.

When the currents deviate from the maximum torque angle, they can be written as

$$\begin{aligned} i_\alpha(\theta) &= -I_p \sin(\theta - \delta) \\ i_\beta(\theta) &= I_p \cos(\theta - \delta) \end{aligned} \tag{10.12}$$

where δ is the angular deviation. Substituting (10.9) and (10.12) into (10.7) and simplifying gives

$$T = K_p I_p \cos(\delta) \tag{10.13}$$

As δ moves away from zero in either the positive or negative direction, the torque produced decreases. This relationship confirms the fact that (10.11) describes the maximum torque.

To identify how the direct and quadrature currents play a role in torque production, it is necessary to substitute the individual equations in (10.6) into (10.7). Doing so, leads to the relationship

$$T = K_p i_q(\theta) \tag{10.14}$$

which agrees with the earlier statement that only the component of the stator current and its magnetic field that is along the rotor's quadrature axis leads to torque production. The direct component produces no torque.

Substitution of (10.12) into (10.5) provides yet further insight,

$$\begin{aligned} i_d(\theta) &= I_d = I_p \sin(\delta) \\ i_q(\theta) &= I_q = I_p \cos(\delta) \end{aligned} \tag{10.15}$$

The direct and quadrature currents are not actually functions of position θ. They are constant values, I_d and I_q respectively, dependent only on the angular offset δ from maximum torque angle. Their variation with respect to angular position disappears because the *d-q* axis where they are defined is stationary with respect to the torque producing rotor permanent magnet field. The results in (10.15) confirm the results shown in (10.14), (10.13), and (10.11). By expressing the motor currents in the *d-q* coordinate system as shown in (10.15), control of the amplitude and phase of the motor currents is determined by controlling the two scalar quantities I_d and I_q, both of which have units of Amperes.

A fundamental consequence of (10.15) and (10.12) is that the amplitude of the current space vector is equal to the peak phase current amplitude in both the α-β and d-q coordinate systems, *i.e.*,

$$I_p = \sqrt{I_d^2 + I_q^2} = \sqrt{i_\alpha^2(\theta) + i_\beta^2(\theta)} \tag{10.16}$$

Since the motor I^2R losses per phase are equal to $I_p^2 R_{ph}/2$ when the currents are sinusoidal, the presence of a direct axis current component diminishes motor efficiency as it decreases $i_q(\theta) = I_q$ and the resulting motor torque as per (10.16) and (10.14) respectfully. However, it does not change I_p, and therefore motor losses remain fixed. This fact explains why vector control algorithms typically act to drive $i_d(\theta) = I_d$ to zero and dynamically control I_q to meet the load torque requirement at the desired speed. Doing so maximizes energy conversion efficiency. The primary exception to this control approach is when implementing flux or field weakening control. In field weakening, the direct current component $I_d = I_p \sin(\delta)$ is driven negative, which implies that δ is driven negative. In doing so, the resulting phase currents are advanced in phase relative to the back EMFs (10.9), thereby making it easier to establish desired current amplitudes as the back EMF amplitudes approach or exceed the amplitude of the supply voltage used to drive the motor.

In summary, the Park transformation provides an alternative perspective and insight into motor operation. Through it, motor operation is seen from the perspective of an observer fixed on the rotor with the stator moving around it.

10.3 Three Phase Motors

Clark Transformation

As described in the previous section, the Park transformation maps stator currents into equivalent currents oriented in space relative to the rotor permanent magnet field. In the two phase case, the phase A and B windings coincided with the stator α- and β-axes respectively. In the three phase case as shown in **Fig. 10-6**, the α-axis coincides with the phase A winding, but the phase B and C phase windings are oriented 120 °E away from each other and from the phase A winding and neither coincides with the β-axis. Current flow in the phase B and C windings creates magnetic fields that have components in both the α and β directions as depicted in **Fig. 10-7**. Therefore, the effective space vector current components with respect to the α-β axes can be written as

$$\begin{aligned} i_\alpha(\theta) &= i_a(\theta) - i_b(\theta)/2 - i_c(\theta)/2 \\ i_\beta(\theta) &= 0\, i_a(\theta) + \sqrt{3}\, i_b(\theta)/2 - \sqrt{3}\, i_c(\theta)/2 \end{aligned} \tag{10.17}$$

where $i_a(\theta)$, $-i_b(\theta)/2$, and $-i_c(\theta)/2$ are components directed along the α-axis and $\sqrt{(3)}\, i_b(\theta)/2$ and $-\sqrt{(3)}\, i_c(\theta)/2$ are components along the β-axis. These expressions